MODELLING CROWD LOAD FOR FLOOR VIBRATION ANALYSIS

Javier Fernández[1], Lutz Hermanns[2], Enrique Alarcón[3], Javier Cara[4]

1: SINEX, S.A. Javier Fernández
Sociedad de Investigación Estudios y Experimentación, S.A.
c/ Espronceda, 34, 1º int.
28003 Madrid
ingeniería@sinex.es

2: CEMIM. Lutz Hermanns
Centro de Modelado en Ingeniería Mecánica
c/ Jose Gutierrez Abascal, 2
28006 Madrid
lhermanns@etsii.upm.es

3: ESCUELA TECNICA SUPERIOR INGENIEROS INDUSTRIALES. Enrique Alarcón
Universidad Politécnica Madrid
Departamento de Estructuras y Construcciones Industriales
c/ Jose Gutierrez Abascal, 2
28006 Madrid
enrique.alarcón@upm.es

4: ESCUELA TECNICA SUPERIOR INGENIEROS INDUSTRIALES. Javier Cara
Universidad Politécnica Madrid
c/ Jose Gutierrez Abascal, 2
28006 Madrid
fjcara@etsii.upm.es

ABSTRACT

The dynamic floor loads induced by crowds in gymnasium or stadium structures are commonly modelled by superposition of the individual contributions using reduction factors for the different Fourier coefficients. These Fourier coefficients and the reduction factors are calculated using full scale measurements.
Generally the testing is performed on platforms or structures that can be considered rigid, such that the natural frequencies are higher than the frequencies of the spectator movement.
In this paper we shall present the testing done on a structure that used to be a gymnasium as well as the procedure used to identify its dynamic properties and a first evaluation of the so-called "group effect".

1. INTRODUCTION

The interest for modelling of human actions acting on structures has been recurrent since the first accidents on suspension bridges in the nineteenth century like Broughton (1831) in the U.K. or Angers (1850) in France. The use of new materials allowing the design of slender structures, the simultaneous interest in the structural serviceability performance and accidents such as during the opening ceremony of the London Millenium Footbridge (10 June 2000) made it mandatory to carry on and in-depth analysis of the equivalent actions to be used in the numerical analysis of structures.

One of the first summaries was due to Bachman [1] et al. where the modelling of individual loads was fruitfully studied. One of the most influential research, conducted by Lenzen and Murray[2] as early as 1969, suggested the use of the so-called "heel drop test" for assessing the vibration susceptibility of light floors under walking loads. Although the general applicability of their results has been questioned [3] its influence on National Codes (like the current Spanish "Código Técnico de la Edificación") has been extensive.

Current research authors are Ebrahimpour, Pernica, Allen, Ellis, etc. (summary of their papers can be found in reference [4]). More recently interesting contributions are due to Ellis and Ji [5] and Sim [6]. Also important are European research projects [7] and [8]. The publication of SCI Guide P354 [9] incorporating new results such as the reduction factors for the Fourier coefficients representing the crowd activities (point 3.1.3) has been of particular interest and is the main motivation for this paper.

Our theoretical research work on this topic started 20 years ago when we had to analyze several floors in gymnasiums used for athletes training for the 1992 Olympics. Since then we continuously improve our testing equipment and recently in the framework of a research project funded by the Spanish Ministry of Science and Innovation [10] we embarked in a systematic theoretical and experimental analysis of different existing structures such as the building described below. In particular we were interested in the lack of coordination in groups of people jumping and tried to see the possibility of checking the above-mentioned coefficients in article 3.1.3 of SCI Guide P354.

In point 2 we shall present some details of one of the structures tested as well as a description of the equipment used and the types of actions the structure was subjected to.

In point 3 we collect the main dynamic properties of the structure. These were obtained after applying the so-called Stochastic Subspace Identification (SSI) which is a method relatively popular in system identification. The estimated modal properties will be used in this paper for separating the action definition from the structural resonance contributions.

Point 4 contains some details of the different crowd densities and the registered accelerograms.

Finally in point 5 we shall describe a couple of methods to identify the Fourier coefficients of the total load by separating the load contribution from the structural resonance in the acceleration time histories.

2. TEST STRUCTURE

The test we shall present was performed on a stand-alone building, formerly used as a gymnasium, that is a part of the School of Industrial Engineering of the Technical University of Madrid. The structure was finished in the 1950's and, unfortunately, the original as-built drawings are not available.

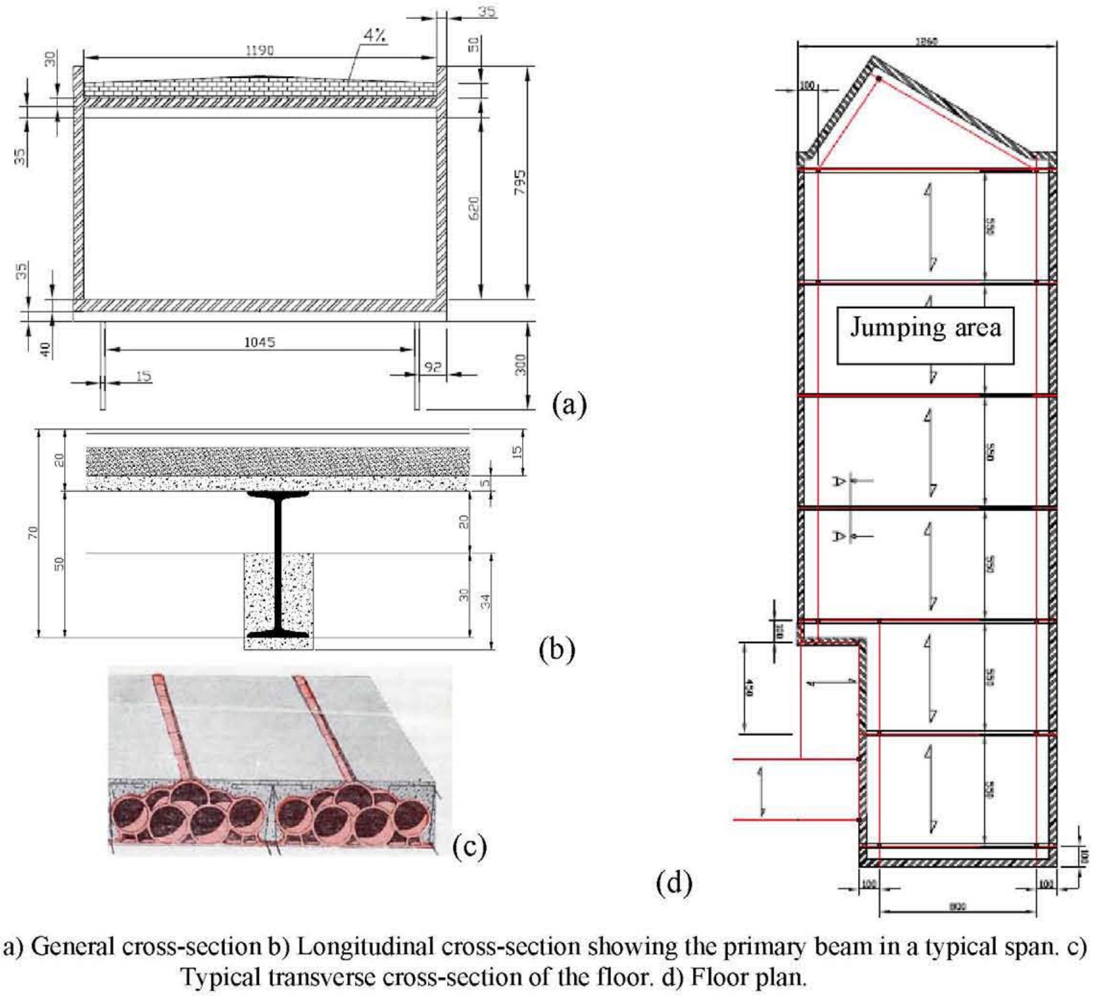


Figure 1. a) General cross-section b) Longitudinal cross-section showing the primary beam in a typical span. c) Typical transverse cross-section of the floor. d) Floor plan.

Figure 1a) shows a cross section of the structure. There is a rectangular closed frame supported on two steel columns. Figure 1b) shows the connection of the floor to the main transverse beam. The beam is a standard steel profile supporting the mixed ceramic-concrete floor through to a concrete block that, apparently, was cast in-situ simultaneously with a concrete slab over the steel profile. That way, structural continuity between adjacent floor spans is achieved. Figure 1c) shows the floor cross section where ceramic pieces were connected by concrete and partial reinforcement forming semi-beams that were distributed between the primary beams. Then complementary reinforcement was installed especially to provide bay continuity and concrete was cast over the system to get the final resisting part of the floor.

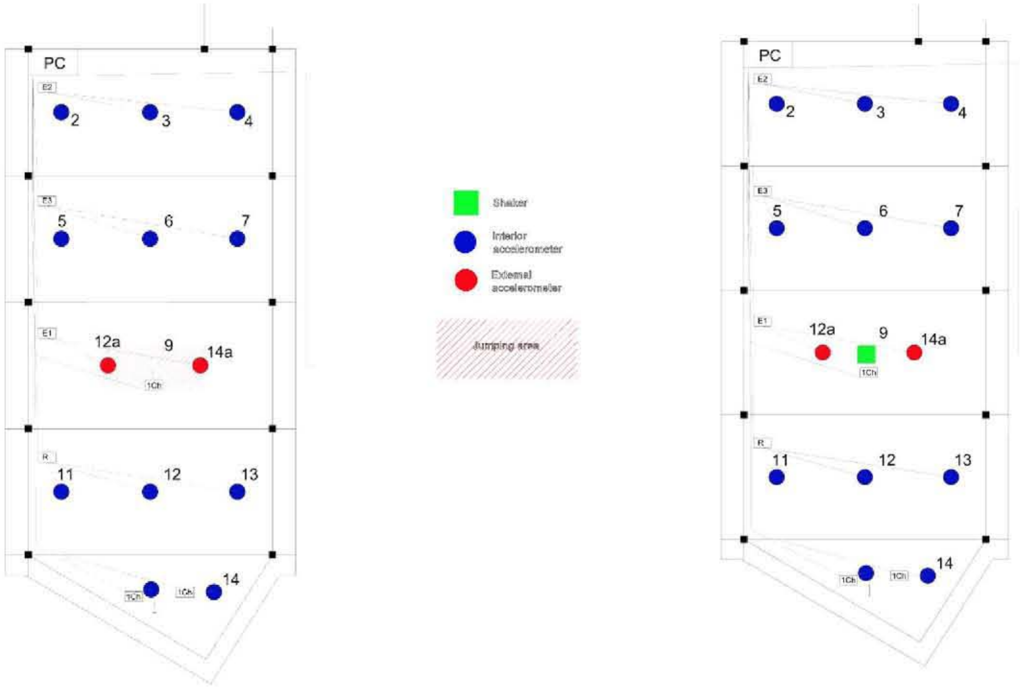
Sand layers and different pavement finishing complete the floor. The upper floor has a similar structure while the vertical partitions are brick walls. Figure 1d) shows the building floor plan.

(a) (b) (c)

Figure 2. External photographs of the structure

The photograph in Figure 2a) shows a general view of the structure which is not simple, as shown in Figure 2b) picturing a large window or in Figure 2c) where an access stair seems to offer a fixed point in horizontal displacements to the second bay.

To analyze this structure subjected to different actions (ambient vibration, electrodynamic shaker, random walk, dance, jumps,etc) we studied several configurations. A couple of those are shown in Figure 3.



(a) (b)

Figure 3. Accelerometer configurations used to register the structural response a) Crowd load configuration b) Shaker configuration

Configuration of the Figure 3a) was used to analyze the structural response under a crowd load and Figure 3b) shows the action of an electrodynamic shaker.

3. IDENTIFIED DYNAMIC PROPERTIES

The typical time-domain stochastic system identification methods work on a discrete time state-space modal

$$\begin{aligned} x_{t+1} &= Ax_t + Bu_t \\ y_t &= Cx_t + Du_t \end{aligned} \tag{1}$$

where

- t denotes the sampling instant with constant sampling time Δt;
- $y_t \in R^{n_0}$ is the measured output vector;
- $u_t \in R^{n_i}$ is the measured input vector;
- $x_t \in R^{n_s}$ is the state vector;
- $A \in R^{n_s x n_s}$ is the transition state matrix describing the dynamics of the system;
- $B \in R^{n_s x n_i}$ is the input matrix;

- C $\in R^{n_0 x n_s}$ is the output matrix, which is describing how the internal state is transferred to the output measurements y_t;
- D $\in R^{n_0 x n_i}$ is the direct transmission matrix;
- The noise vectors comprise immeasurable vector signals assumed to be zero-mean, white vector sequences with covariance matrices

$$w_t \to N(0,Q) \qquad v_t \to N(0,R) \tag{2}$$

Equation (1a) is known as the state equation and equation (1b) is known as the observation equation. In the case of output-only vibration testing, only the responses of a structure is measured, while the input sequence u_t remains unmeasured. Thus, Equation (1) results in a purely stochastic system:

$$\begin{aligned} x_{t+1} &= Ax_t + w_t \\ y_t &= Cx_t + v_t \end{aligned} \tag{3}$$

From the eigenvalues of matrix A it is possible to get the natural frequencies and the modal damping ratios. The eigenvectors can be obtained from matrix C.

Using the subspace method the following properties of the test structure were obtained:

	Frequency (Hz)	Damping ratio(%)
Horizontal Modes	1.15	1.37
	2.27	1.01
	3.23	1.68
Vertical Modes	4.39	1.94
	5.6	1.72
	5.74	1.6
	6.75	1.32
	8.54	1.91
	10.85	2.33

Table 1. Natural frequencies and damping ratios

Table 1 shows that the damping ratios are relatively low and the natural frequencies can be excited by harmonics of jumping rhythm over 2 Hz.

In addition to the accelerometers, a laser was placed under the excited bay (in the jumping area, see Figure 1d) in order to measure vertical displacements.

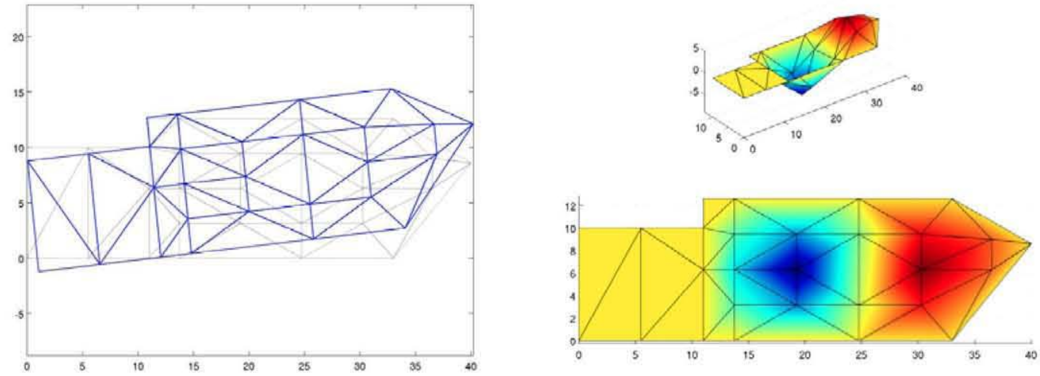


Figure 4. a) First horizontal mode b) 4^{th} vertical mode

Figure 4a) shows the fixed point effect of the access external stair producing a global horizontal rigid-body tipe rotation of the floor.

Figure 4b) shows the continuity effect between neighbour bays reflected in the vertical displacements around the excited area which is an effect that was clearly perceived during the tests (the rectangular area on the left could not be instrumented).

4. CROWD DISTRIBUTION.

The jumping area is 12 m^2. The jumpers were distributed in several groups of three people. Combining these groups, tests with different number of people were carried out: 1 person (Figure 6a), 6, 12 (Figure 6b), 18 (figure 6c), 24 and 30 people. All jumpers were weighted before starting the tests in order to assess the total load in each set of jumps. Figure 5 and Table 2 illustrates an example of a couple of these group combinations. The grey rectangle is the jumping area.

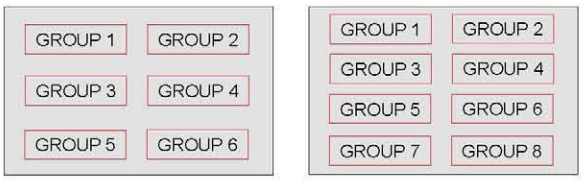


Figure 5. a) Combination 1: 18 people. b) Combination 2: 24 people.

	Group							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Weight (Kg)	218	213	183	215	242	228	231	224
Total weight (Kg)	**1299** (Combination 1)						455	
	1754 (Combination 2)							

Table 2. Combination weights

The jumping was coordinated using a musical beat at selected frequencies. The vertical acceleration of the centre of the floor was recorded with the accelerometers. Photographs of the people jumping and an acceleration time-history are shown in Figure 6 and Figure 7 respectively.

(a) (b) (c)

Figure 6. Photographs of people jumping

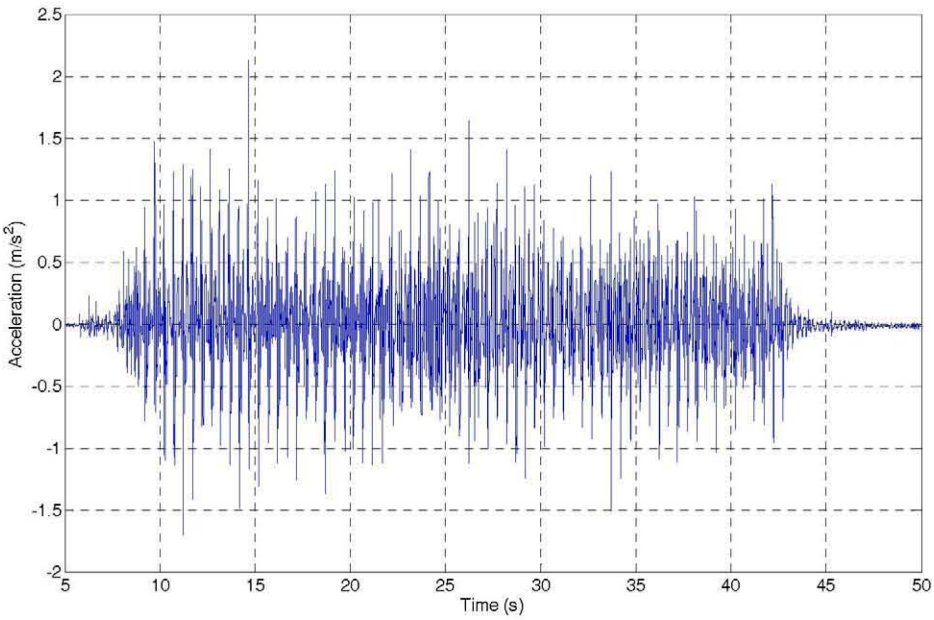


Figure 7. Acceleration time-history.

5. IDENTIFIED FOURIER COEFFICIENTS

In order to obtain a representation of the acting load in the same form as the one proposed by SCI P354 guide [9] we decided to follow the procedure explained by Ellis [5].

The global force is:

$$F(t) = W[1 + \sum_{j=1}^{3} \alpha_j sen(\omega_j t + \phi_j)] \tag{4}$$

where W is the total acting load, ω_j is j times the jumping frequency, and α_j is adjusted to the number of jumping people to reflect the group effect.

Figure 8 shows the displacement time-history of a point under the jumping area for the case of a crowd jumping at 2 Hz.

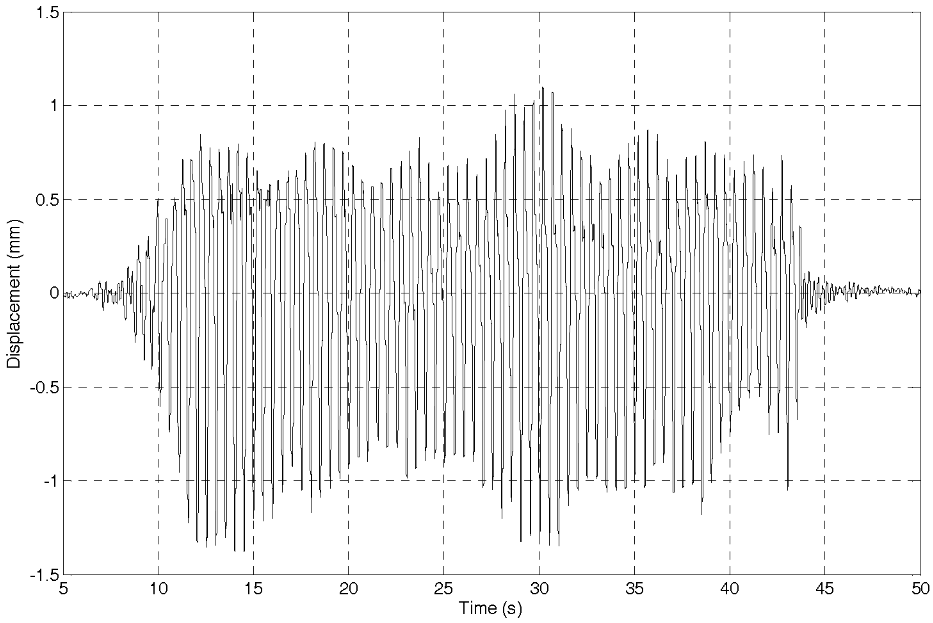


Figure 8. Displacement time-history

Following Ellis [5] suggestion, we band-pass filtered the signal around frequencies 2, 4 and 6 Hz, obtained the RMS value, multiplied the result by $\sqrt{2}$ and divided the result by the static displacement of the group. We repeated the process with different successive pieces of 5 seconds of duration and plotted the results against the number of people on a log-log plot.

Figure 9 shows the regression line for a power relationship that produced results close to the results proposed in ref [5,9] in spite of the 2nd and the 3rd Fourier components being close to the natural frequencies.

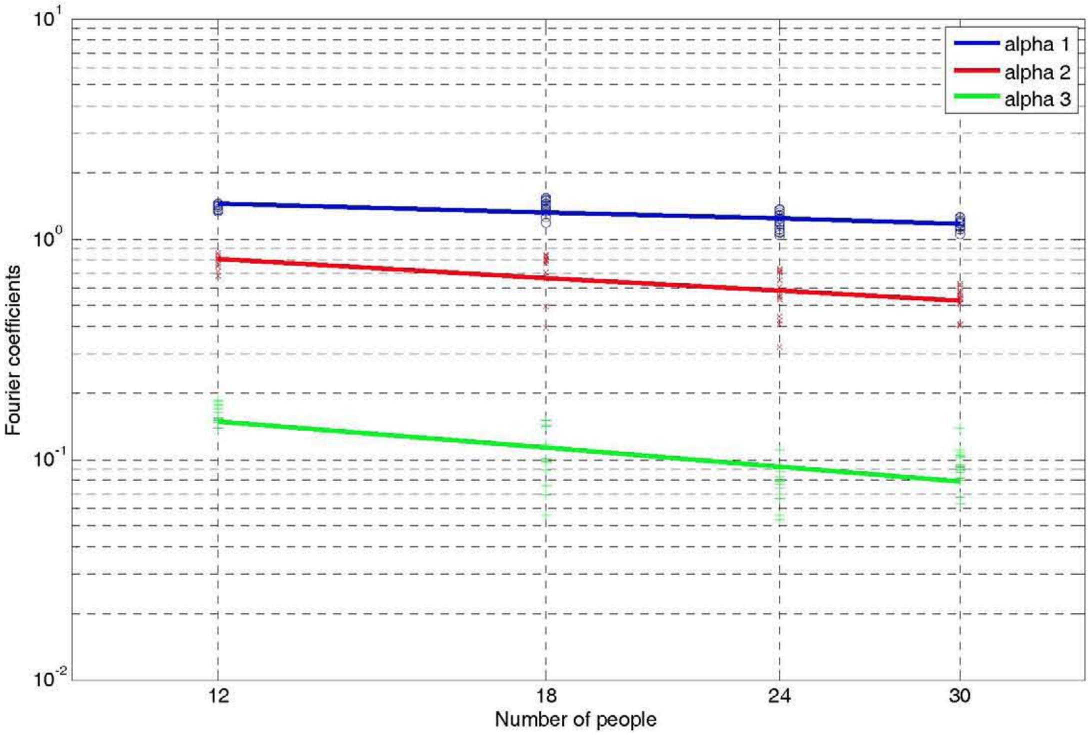


Figure 9. Fourier Coefficients-Number of people (jump frequency : 2Hz)

In order to get rid of the possible resonance we used again the previous work with SSI method.

In equations (3), the input is implicitly modelled by the noise terms w_t , v_t. However the white noise assumptions for these noise terms cannot be omitted and (2) remain still applicable in equation (3). The consequence is that if this white noise assumption is violated, for instance if the input contains also some dominant frequency components in addition to white noise, these frequency components will appear as poles of the state matrix A.

But the state of a system is not unique: given the linear time-invariant (LTI) system (3), we can transform the state x_t into z_t as follows

$$x_t = Tz_t \tag{5}$$

where T is the transformation matrix. Replacing this condition into (3) and pre-multiplying by T^{-1}

$$z_{t+1} = A_0 z_t + T^{-1} w_t$$
$$y_t = C_0 z_t + v_t \tag{6}$$

where

$$A_0 = T^{-1} A T \qquad C_0 = CT \tag{7}$$

This state representation yields the same dynamic relation between input and y_t, that is, the same input-output behaviour that (3). On the other hand, the eigenvalue problem of matrix A is stayed as

$$AV = VD \tag{8}$$

where D is the eigenvalue matrix and V is the eigenvector matrix. If we choose V = T, this implies that A_0 = D, so the matrix A_0 would a diagonal matrix, which diagonal elements are complex conjugate values and each pair represents a vibration mode. We can represent A_0 as:

$$A_0 = \begin{vmatrix} \lambda_1 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & \lambda_1^* & \dots & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_i & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & \lambda_i^* & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & \lambda_{n/2} & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & \lambda_{n/2}^* \end{vmatrix} \tag{9}$$

On the other hand, the Kalman filter (see Appendix A) allows us to compute the "optimal" state sequence

$$\begin{bmatrix} x_1^{\ 0} & x_2^{\ 1} \cdots & x_t^{\ t-1} \cdots & x_N^{\ N-1} \end{bmatrix} \tag{10}$$

which can be converted into a state sequence in modal form by mean of the transformation matrix T = V

$$z_t^{t-1} = V^{-1} x_t^{t-1} \Rightarrow \begin{bmatrix} z_1^{\ 0} & z_2^{\ 1} \cdots & z_t^{\ t-1} \cdots & z_N^{\ N-1} \end{bmatrix} \tag{11}$$

The corresponding matrix for these states is A_0 (eq. (9)). So, selecting row i and i + 1 we obtain the states for mode i,

$$z_t^i = \begin{bmatrix} 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 1 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & \dots & 0 & 0 & \dots & 0 & 0 \end{bmatrix} [z_1^0 \quad z_2^1 \quad \cdots \quad z_t^{t-1} \quad \cdots \quad z_N^{N-1}] \tag{12}$$

and using the observation equation we can obtain the response due to mode i.

$$y_t^{\text{mode i}} = C_0 z_t^{\text{mode i}} \tag{13}$$

Repeating this procedure for each mode, the total response is

$$y_t = y_t^{\text{mode 1}} + y_t^{\text{mode 2}} + \cdots + y_t^{\text{mode n/2}} + v_t \tag{14}$$

so we can separate modes due to jumping and structural modes.

$$y_t = y_t^{jump} + y_t^{struc} + v_t \tag{15}$$

Once the values corresponding to the jumping frequency are separated, it is possible to have the acceleration time-history y(t) decomposed in a part due to the structural resonance y_{mod} and the other one due to the jumping load y_{arm} (Figure 10).

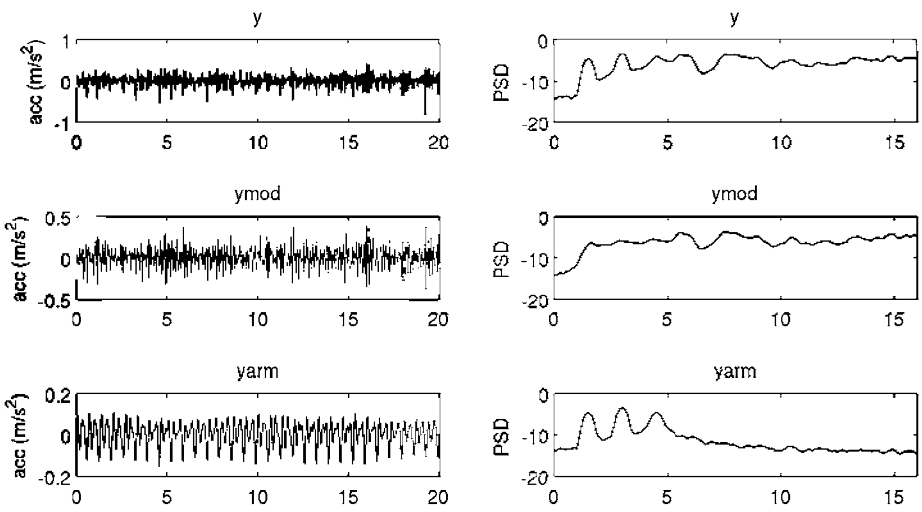


Figure 10. Decomposed acceleration.

From the acceleration time-history due to the jumping load (y_{arm}) and applying the method described previously [5], new values of Fourier coefficients have been obtained. In Figure 11 these new results are compared with the coefficients assessed with the total acceleration for a jump frequency of 1.5 Hz.

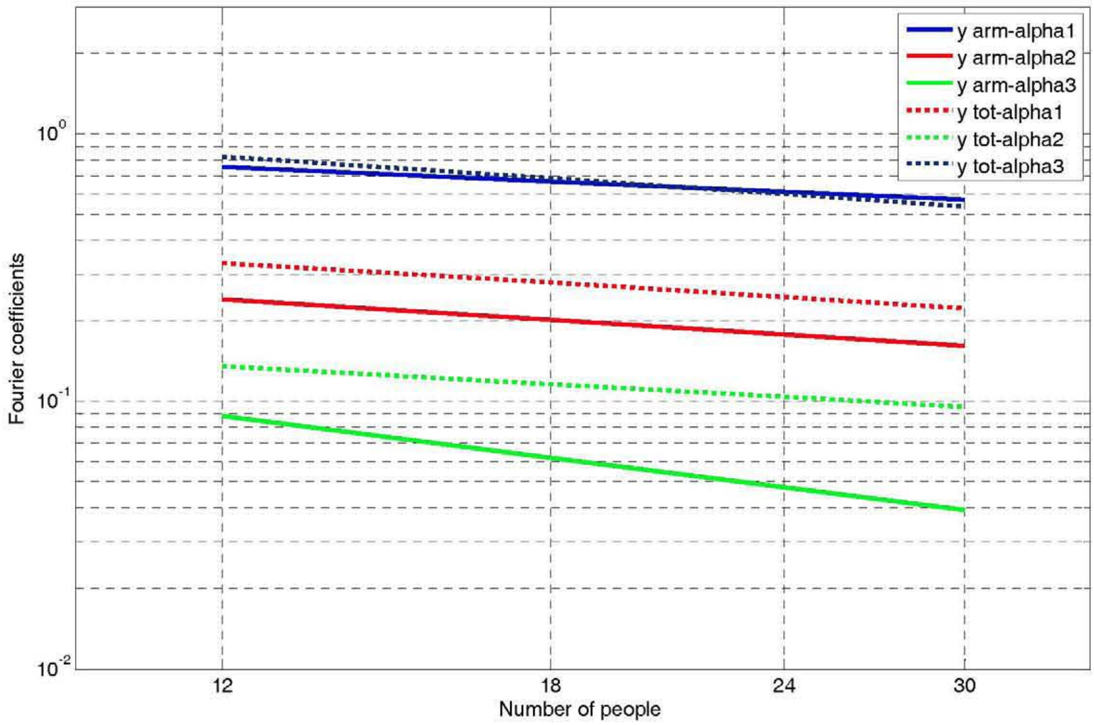


Figure 11. Fourier Coefficients-Number of people (jump frequency : 1.5 Hz)

The figure shows that first coefficient values of the acceleration due to jump load are very close to the values corresponding to the total acceleration. However respect to the second and third coefficient values, the differences are higher, because these Fourier components are close to the natural frequencies of the floor.

6 CONCLUSIONS

The paper has studied the numerical modeling of crowd induced loads from a set of vibration data recordings obtained during tests on a real structure under controlled conditions.

The Fourier coefficients of the total load have been calculated. The results are similar to the results proposed in ref [5,9] in spite of the 2nd and the 3rd Fourier components being close to the natural frequencies. The results demonstrate the “group effect”, if the number of people is increased, the coefficient values decrease.

Using the SSI method the acceleration can be decomposed in a part due to the structural resonance and the other one due the jumping load. If the procedure for calculating the coefficients is applied to the acceleration under jump loads, the results of the first coefficients are close to the results of the total acceleration, but the there is difference in the results of the second and third coefficients.

7 ACKNOWLEDGMENTS

Authors would like to thank the Spanish Ministry of Science and innovation for sponsoring this work through Research Project BIA 2008-0489.

REFERENCES

[1] Bachmann,H. and Ammann,W. (1987) "Vibrations in Structures Induced by man and machines" IABSE

[2] Lenzan,K.H. and Murray,T.M.(1969):"Vibration of steel Beam Concrete Slab Floor Systems",Repp.29. Depart.Civil Eng. University of Kansas.

[3] Ellis,B.R.(2000) "On the response of long span floors to walking loads generated by individuals and crowds". The structural engineer,**78**(10), May, 17-25.

[4] Smith,J.W. "Human-induced vibrations" Chapt. 7 in "Dynamic Loading and Design of Structures" Ed. A.J.Kappos, Spon Press (2002)

[5] Ellis,B.R. and Ji,T (2004) "Loads generated by jumping crowds;numerical modelling". The structural Engineer **82**(17), 35-40

[6] Sim J.H.H. (2006) "Human-structure interaction in cantilever grandstands" Ph.D. Thesis. University of Oxford .

[7] G. Sedlacek, C. Heinemeyer,C. Butz, B. Volling , RWTH, Aachen ; P.Waarts, F, Van Duin, TNO Institute of Technology; S. Hicks, P. Devine, The Steel Constructiion Institute; T.Demarco Profilarbed. "Generalitation of criteria for floor vibractions for industrial, office, residential and public building and gymnastic halls." EUR 21972 EN-2006

[8] Feldmann M. et al: "Design of floor structures for human induced vibrations" Joint report prepared under the JRC-ECCS cooperation Agreement for the evolution of Eurocode 3. EUR 24084 EN-2009

[9] Steel Construction Institute (2007) "Design of floors for vibration :A New Approach". SCI P 354.

[10] Ministerio de Ciencia e Innovación "Vibraciones estructurales inducidas por la actividad humana". BIA 2008-04089.